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Air defense planning for an area with the use of very short range air defense sets*

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Abstract. This paper presents a heuristic method of planning the deployment of very short-range anti-air missile and artillery sets (VSHORAD) around an area ('protected area') in order to protect it. A function dependent on the distance between the earliest feasible points of destroying targets and the centre of the protected area was taken as an objective function. This is a different indicator from those commonly used in the literature, and based on the likelihood of a defense zone penetration by means of an air attack (MAA): the kill probability of the MAA and the probability of area losses. The model constraints resulted directly from the restrictions imposed by real air defense systems and the nature of the area being defended. This paper assumes that the VSHORAD system operates as a part of a general, superordinate air defense command and control system based on the idea of network-centric warfare, which provides the VSHORAD system with a recognized air picture, air defense plans, and combat mission specifications. The presented method has been implemented. The final part of the paper presents the computational results.
Keywords: optimal planning, air defense system, area installation protection, deployment of very short range anti-air missile and artillery sets (VSHORAD)
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1. Introduction

Modern means of air attack (MAA) can penetrate a defended air space and territory and destroy high-value economic and defense installations. The primary

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MAA countermeasures include anti-aircraft weaponry sets (AAW). AAW form a critical component of any air defense system used to protect strategic discrete and area installations. AAW can be deployed as short-range air defense (SHORAD) and very short-range air defense (VSHORAD) systems around strategic installations. A VSHORAD system usually features the following components:

— anti-aircraft missile and artillery (AAMA) sets;
— short-range radar unit;
— air defense system control unit, which exchanges data with a superordinate air defense command system.

This paper assumes that an air defense system control post has been connected to a national air defense system. This means that the control post should receive:

— recognized air picture for the region and the approach directions for the protected area;
— air defense plans from a superordinate air defense command system unit.

Air defense systems based on AAMA sets can be effectively used at the stage of repelling air attacks and effectively defending a protected installation, assuming that the VSHORAD deployment is properly located at the air defense deployment planning stage. Air defense deployment planning is a process which requires the consideration of many factors. These factors include: terrain properties which may affect the local radar unit's detection zone, and the VSHORAD kill zones. Reference sources show a definite trend toward the inclusion of as many of these factors as possible in the air defense deployment planning process [3]. The authors believe that this approach is very reasonable when applied to a different problem, which is weapon-target assignment (WTA) in real time at the air raid repelling stage [5, 6, 7, 8]. In the process of planning AAMA set deployment, many highly uncertain factors exist, including the uncertainty of (i) enemy MAA type, (ii) enemy MAA ordinance, (iii) enemy air raid composition, (iv) enemy MAA flight trajectory, (v) enemy MAA tactics, (vi) enemy reconnaissance of the protected area, and much more. It was thus decided to assume simplified models for the VSHORAD and the local radar's identification zones.

Other and no less important factors which affect the operating capabilities of VSHORAD systems include the quality of the WTA methods for the individual AAMA sets and the AAMA set fire control methods. While these two categories are partially interrelated, this paper focuses only on the first one, which includes planning the deployment of AAMA sets within an air defense system for a protected area. The other problem is discussed in most of the applicable references [5, 6, 7, 8].

The paper assumes that the AAMA set deployment planning stage follows a field reconnaissance stage. The outcome of the reconnaissance stage is the determination of the feasible potential deployment positions of the AAMA sets. A specific position may be feasible for AAMA set deployment due to favourable terrain conditions and/or the installation of essential engineering facilities.

In short, the problem of AAMA set deployment in an air defense system for a protected area is to deploy a defined number of AAMA sets in optimally selected locations around a defined area or a set of points referred to as 'protected objects', with due consideration of the properties of the surrounding terrain and the characteristics of the kill zones of different targeting channels. Attack and defense are two aspects of the same problem; which is why a VSHORAD deployment planning analysis should consider different variants of enemy air operations.

2. Model of the protected installation

One assumption in this paper is that the protected installation is a protected area. An area comprises a protected installation ('protected object'), the defense or economic value of which is distributed over a certain physical area or at specific points in the physical area.

The basic assumptions are that the protected area Ω is bounded by a polygon and the elevation values of the points in the protected area are approximately the same. The protected area do not have to precisely coincide with the actual area of the protected object that the former represented. Selected vertices of the polygon defining the protected area determine the operating result of a targeting channel deployment planning algorithm, i.e. the selection of the best deployment points for the targeting channels.

The following designations are used:

B — set of vertices for the protected area, corresponding to a polygon, with:

$$\boldsymbol{B} = \left\{ B_i = (x_i^b, y_i^b, h_i^b) \in \mathsf{R}^3, i = 1, \ldots, I \right\}, \tag{1}$$

whereas: $B_i = (x_i^b, y_i^b, h_i^b)$ — a polygon vertex;

x_i^b, y_i^b, h_i^b — Cartesian coordinates of the polygon vertex *i*;

I — total number of polygon vertices.

Given this assumption, one can write:

$$\forall i, j = 1, \ldots, I \; h_i^b \approx h_j^b. \tag{2}$$

Given (1) and (2), the set Ω is a convex combination of the set of polygon vertices ***B***:

$$\Omega = \left\{ (x, y) \in \mathsf{R}^2 : (x, y) = \sum_{i=1}^{I} \alpha_i (x_i^b, y_i^b), \alpha_i \in [0,1], \sum_{i=1}^{I} \alpha_i = 1, (x_i^b, y_i^b) \in \boldsymbol{B} \right\}. \tag{3}$$

As explained in the abstract, the function of the installation value is determined for the protected area. The function of the installation value can be a continuous planar distribution of value density defined for the area of a convex protected area Ω, or a distribution of value points across the same area.

When a continuous planar distribution of value density is assumed, the function of value density *w*: $\mathsf{R}^2 \rightarrow \mathsf{R}$ is determined for each point on the plane to meet the following condition:

$$w(x,y)\begin{cases} = 0 \text{ dla } (x,y) \notin \Omega \\ \geq 0 \text{ dla } (x,y) \in \Omega. \end{cases} \tag{4}$$

When a distribution of value points in the protected area is assumed, the elementary protected point objects comprising the protected area (x_a, y_a) are determined. Let $\boldsymbol{A} = \{1,\ldots, A\}$ be a set of numbers for the elementary protected point objects in the protected area; each elementary protected point object has the coordinates (x_a, y_a) with the value w_a.

In both cases, the coordinates of the centre of the protected area (x_s, y_s) are determined, where this centre is construed as the centre of distribution of the values in the protected area.

In the continuous planar distribution case, the following formula is obtained [9]:

$$x_s = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xw(x,y)dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w(x,y)dxdy}, \quad y_s = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} yw(x,y)dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w(x,y)dxdy} \tag{5}$$

In the discrete distribution case, the following formula is obtained [9]:

$$x_s = \frac{\sum_{a=1}^{A} x_a w_a}{\sum_{a=1}^{A} x_a}, \quad y_s = \frac{\sum_{a=1}^{A} y_a w_a}{\sum_{a=1}^{A} x_a} \tag{6}$$

3. Model of the air defense system for the protected area

3.1. Model of the AAMA targeting channels

As explained above, the work assumes that the protected area is defended by a VSHORAD system, comprising:

— anti-air missile and/or artillery sets (AAMA sets);
— short-range radar unit;

— air defense system control post, which exchanges data with a superordinate air defense command system.

Due to practical considerations, the AAMA sets can differ in characteristics. Three AAMA set types are assumed as feasible for protecting the protected object.

The first two types are basic AAMA set types:

— anti-aircraft artillery set (AAAS);
— surface-to-air missile set (SAM).

The third AAMA set type is complex and created by combining the two basic AAMA set types specified before.

Each AAMA set type features the following attributes:

— the AAMA set effective range d_t, with t as the AAMA set type number ($t = 1,...,T$), construed as the maximum kill zone radius of the AAMA set [1, 2, 3];
— the mean AAMA set kill effectiveness s_t, with t as the AAMA set type number ($t = 1,...,T$); the mean kill effectiveness is construed as the mean kill probability in the kill zone [3, 4, 5, 7].

The attributes listed above are averaged characteristics. The reference literature dedicated to the problems of AAMA set placement and WTA [1, 2, 3, 4, 5, 6, 7, 8] considers many detailed characteristics, including (and not limited to):

— spatial distribution of target kill probability;
— terrain properties, mainly a topographical model of the terrain;
— target duration in the kill zone(s);
— overlap ratio between the kill zones;
— layout of the exposed zones of protected objects;
— shape and size of the detection zone of the local air defense radar unit.

The kill range and effectiveness of the third AAMA set type of (AAMAS) depend on the values of the respective attributes of the first and second AAMA set types, as shown in the formulas below:

$$d_3 = \max\{d_1, d_2\}, \quad s_3 = \alpha[1-(1-s_1)(1-s_2)], \tag{7}$$

with $\alpha \in (0,1]$, being the actual to ideal kill effectiveness ratio, where the actual kill effectiveness is lower than the ideal one, given the non-feasibility of simultaneous operation of both targeting channels in the third type of AAMA sets.

Quantity limits are imposed: a maximum number is defined for the AAMA sets of specific types that can be used to defend the protected object. K_t is the maximum number of the AAMA sets of type number t ($t = 1,...,T$) which can be used to defend the protected object.

3.2. Model of the AAMA set deployment

A AAMA set can be placed in one of a number of predefined deployment points. The set of feasible deployment points should be defined by field reconnaissance. The following features are considered in this process:

— characteristics of the terrain around each deployment point;
— engineering infrastructure of the terrain.

Only one AAMA set can be deployed in any single deployment point. The following designation is used:

$\overline{\boldsymbol{D}}$ — set of all feasible deployment points;

$$\overline{\boldsymbol{D}} = \left\{ \overline{D}_j = (\overline{x}_j^d, \overline{y}_j^d) \in \boldsymbol{R}^2,\ j = 1, \ldots, \overline{J} \right\}. \tag{8}$$

The formula (8) do not include the third coordinate of the deployment point: the elevation (height above sea level), given the assumption that the elevation values of all deployment points are approximately the same.

Another assumption is that the air defense plan includes a forecast of the enemy's operational (air raid) scenario. Each air raid scenario can limit the set of all feasible deployment points. Another designation is used:

$\boldsymbol{D}$ — the restricted set of potential deployment points (for a specific air raid scenario) selected from the set $\overline{\boldsymbol{D}}$ and considered during an air defense planning task;

$$\boldsymbol{D} = \left\{ D_j = (x_j^d, y_j^d),\ j = 1, \ldots, J \right\} \in \overline{\boldsymbol{D}}, \tag{9}$$

with: $D_j = (x_j^d, y_j^d)$ — potential AAMA set deployment point;
x_j^d, y_j^d — Cartesian coordinates of a potential AAMA set deployment point;
J — number of potential deployment points.

3.3. Model of the MAA air raid

The contents of the air defense plan guidelines (ADPG) should be provided by a superordinate air defense command unit and comprise guidelines relevant to a specific forecast for an enemy air raid. An assumption is made that the ADPG contained at least one air raid direction.

Each single air attack direction has an azimuth expressed in degrees (with the azimuth determined clockwise from the north).

In the case of multiple air raid directions, each air attack direction is characterised by its azimuth α_n and air raid direction probability $p_n (0 \le p_n \le 1, \sum_{n=1}^{N} p_n = 1,$ whereas N is the number of potential air raid directions.

The MAA air raid model adopted here includes a certain simplification: all air raid directions cross the centre of the protected area, determined by (5) or (6).

The ADPG can also determine the maximum number of AAMA sets of specific types to be used in targeting channel deployment planning. If not, the maximum number should be determined by the VSHORAD commander. The following designation is used:

K_t is the maximum number of AAMA sets of type number t (t = 1,...,T) which could be used in VSHORAD deployment planning.

Given that actual combat mission orders feature azimuth angles α_A^n expressed in degrees and clockwise from the north, the air raid azimuth angles have to be converted for air raid angles expressed in radians relative to the horizontal axis (OX) of the local rectangular coordinate system and indicated counter-clockwise for each air attack direction n.

Degrees are converted to radians with the formula:

$$\alpha_n = \alpha_A^n \frac{\pi}{180} [\text{rad}] \tag{10}$$

with: α_A^n — n value of the air raid azimuth in degrees;
α_n — n value of the air raid angle in radians;
n — sequential number n of the air attack direction.

Air raid angle values β_n are calculated in the local rectangular coordinate system according to these principles:

1. If $0 \le \alpha_n \le \frac{\pi}{2}$, then:

$$\beta_n = \frac{\pi}{2} - \alpha_n. \tag{11}$$

2. Otherwise:

$$\beta_n = (2\pi - \alpha_n) + \frac{\pi}{2}. \tag{12}$$

Fig. 1 illustrates the interrelations of air raid direction, air raid direction unit vector $\boldsymbol{w}_n$, air raid direction azimuth angle α_n and air raid angle β_n.

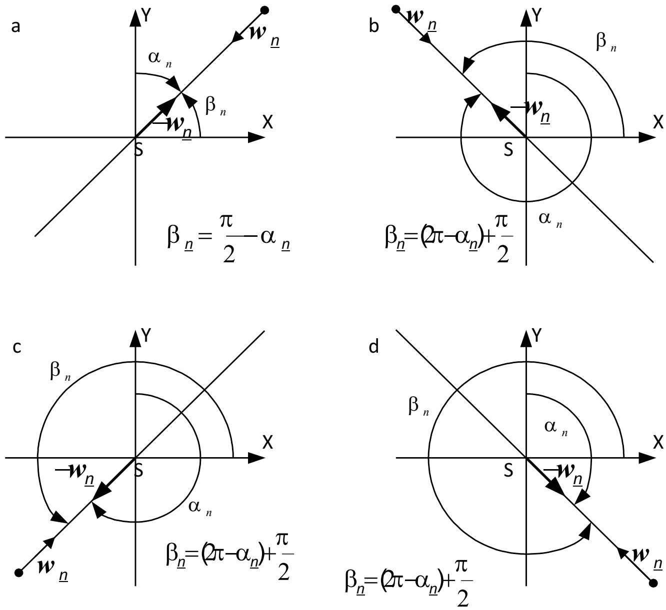


Fig. 1. The interrelations of air raid direction, air raid direction unit vector $\boldsymbol{w}_n$, air raid direction azimuth angle an α_n and air raid angle β_n.

4. Determination of feasible target kill points for the AAMA sets

This section shows how the coordinates are determined for the permitted AAMA set deployment points at which the foreseen air targets can be killed. The determination is based on the models of the protected area, the AAMA sets, and the forecast air raid.

4.1. Determination of air raid direction unit vectors

The air attack direction unit vectors $\boldsymbol{w}_n$

$$w_n(w_x^n, w_y^n) \tag{13}$$

(see thick lines in Fig. 2) help to determine the physical capability of the AAMA sets to kill air targets. Note that the air raid direction unit vectors $\boldsymbol{w}_n$ depend on the air raid angles β_n as follows:

$$w_x^n = \cos\beta_n, \quad w_y^n = \sin\beta_n. \tag{14}$$

Also,

$$\|\mathbf{w}_n\| = \sqrt{\left(w_x^n\right)^2 + \left(w_y^n\right)^2} = 1. \tag{15}$$

4.2. Determination of permitted AAMA set deployment points

To determine the permitted AAMA set deployment points at which the AAMA sets could kill air targets in specific air raid directions (see Fig. 2), an equation is used describing a straight line crossing the centre of the protected area $S = (x_s, y_s)$ and parallel to the vector $\boldsymbol{w}_n = (w_x^n, w_y^n)$. The equation is expressed as follows:

$$\frac{x - x_s}{w_x^n} = \frac{y - y_s}{w_y^n}, \tag{16}$$

which is

$$w_y^n x - w_x^n y - w_y^n x_s + w_x^n y_s = 0. \tag{17}$$

The factors of the general straight line equation (17) are as follows:

$$A_n = w_y^n, \quad B_n = -w_x^n, \quad C_n = -w_y^n x_s + w_x^n y_s. \tag{18}$$

The distance d_j^n of point $D_j = (x_j^d, y_j^d)$ from the air raid direction n is determined by this formula (which, in geometrical terms, is the distance of the point from the straight line):

$$d_j^n = \frac{\left| w_y^n x_j^d - w_x^n y_j^d - w_y^n x_s + w_x^n y_s \right|}{\sqrt{\left(w_x^n\right)^2 + \left(w_y^n\right)^2}}. \tag{19}$$

Given (15), ultimately one gets the following formula:

$$d_j^n = \left| w_y^n x_j^d - w_x^n y_j^d - w_y^n x_s + w_x^n y_s \right|. \tag{20}$$

When $d_j^n \le d_t$ (d_t is the kill range of the AAMA set of type t), the AAMA set of type t could kill air targets in air raid direction n with the AAMA set placed at deployment point D_j.

If $d_j^n > d_t$, then a AAMA set type t target channel could not kill air targets in air raid direction n with the AAMA set placed at deployment point D_j.

The values of d_j^n allow determination of the value of matrix $\boldsymbol{Z}$ that defines the kill capabilities of the targeting channels of all AAMA types at specific deployment points:

$$\boldsymbol{Z} = [z_{jnt}]_{J \times N \times T}, \quad z_{jnt} \in \boldsymbol{R}, \tag{21}$$

whereas:

$$z_{jnt} = \begin{cases} -\infty & \text{gdy} \quad d_j^n > d_t \\ 1 & \text{gdy} \quad d_j^n \le d_t, \end{cases} \tag{22}$$

where $-\infty$ is the lowest negative number assignable to the adopted variable type (the symbol $-\infty$ denotes zero kill capability).

4.3. Determination of the coordinates of the air target kill points for the target channels

This section discusses the method of determining the coordinates of the air target kill points for the AAMA sets capable of killing air targets $(z_{jnt} = 1)$.

The concept for the problem solution is shown in Figs. 2a and 2b. Segment $\overline{O_j^{n,1} O_j^{n,2}}$ is a set of placement points at which AAMA sets are capable of killing air targets.

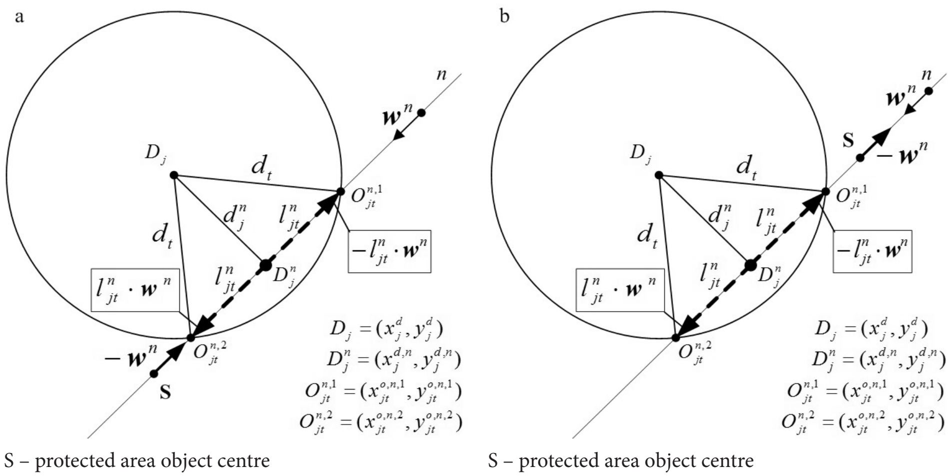


Fig. 2. Illustration of the concept of determining the air target effective kill point for a AAMA set.

First, the coordinates are determined for point $D_j^n = (x_j^{d,n}, y_j^{d,n})$, which is an orthogonal projection of point $D_j = (x_j^d, y_j^d)$ on straight line n.

When point $D_j = (x_j^d, y_j^d)$ is on straight line n, the point coordinates meet the equation of the same straight line:

$$A_n x_j^d + B_n y_j^d + C_n = 0, \tag{23}$$

point $D_j^n = (x_j^{d,n}, y_j^{d,n})$ coincides with point $D_j = (x_j^d, y_j^d)$, which is equal to

$$(x_j^{d,n}, y_j^{d,n}) = (x_j^d, y_j^d). \tag{24}$$

When point $D_j = (x_j^d, y_j^d)$ is not on straight line *n*, the point coordinates do not meet the equation of the same straight line (22):

$$A_n x_j^d + B_n y_j^d + C_n \neq 0. \tag{25}$$

Vector $\overrightarrow{D_j D_j^n}$ has to be perpendicular to vector $\boldsymbol{w}^n$; hence the scalar product of the vectors has to be zero. This gives the following equation:

$$\left(\overrightarrow{D_j D_j^n} \middle| \mathbf{w}_n \right) = 0 \tag{26}$$

addition of the equation of straight line *n* (22) gives this system of equations:

$$\begin{cases} (x_j^{d,n} - x_j^d) w_x^n + (y_j^{d,n} - y_j^d) w_y^n = 0 \\ A_n x_j^{d,n} + B_n y_j^{d,n} + C_n = 0, \end{cases} \tag{27}$$

these are converted to:

$$\begin{cases} w_x^n x_j^{d,n} + w_y^n y_j^{d,n} = w_x^n x_j^d + w_y^n y_j^d \\ A_n x_j^{d,n} + B_n y_j^{d,n} = -C_n. \end{cases} \tag{28}$$

The resulting system of equations gives the determinants:

$$W = \begin{vmatrix} w_x^n & w_y^n \\ A_n & B_n \end{vmatrix}, \quad W_1 = \begin{vmatrix} w_x^n x_j^d + w_y^n y_j^d & w_y^n \\ -C_n & B_n \end{vmatrix}, \quad W_2 = \begin{vmatrix} w_x^n & w_x^n x_j^d + w_y^n y_j^d \\ A_n & -C_n \end{vmatrix} \tag{29}$$

and the following solution:

$$x_j^{d,n} = \frac{W_1}{W}, \quad y_j^{d,n} = \frac{W_2}{W}. \tag{30}$$

Given (15), it is possible to prove that the value of determinant *W* is not equal to zero:

$$W = \begin{vmatrix} w_x^n & w_y^n \\ A_n & B_n \end{vmatrix} = w_x^n B_n - w_y^n A_n = -w_x^n w_x^n - w_y^n w_y^n = -\left[\left(w_x^n \right)^2 + \left(w_y^n \right)^2 \right] = -1. \tag{31}$$

Ultimately, the solution has the following form:

$$x_j^{d,n} = -W_1, \quad y_j^{d,n} = -W_2. \tag{32}$$

Given (14) and (29) in the formulas (32) and (18), the final solution of the system of equations (28) is:

$$\begin{aligned} x_j^{d,n} &= -\sin^2\beta_n\left(x_j^d - x_s\right) + \sin\beta_n\cos\beta_n\left(y_j^d - y_s\right) + x_j^d = \\ &= -\left(w_y^n\right)^2\left(x_j^d - x_s\right) + w_y^n w_x^n\left(y_j^d - y_s\right) + x_j^d, \end{aligned} \tag{33}$$

$$\begin{aligned} y_j^{d,n} &= -\cos^2\beta_n\left(y_j^d - y_s\right) + \sin\beta_n\cos\beta_n\left(x_j^d - x_s\right) + y_j^d = \\ &= -\left(w_x^n\right)^2\left(y_j^d - y_s\right) + w_y^n w_x^n\left(x_j^d - x_s\right) + y_j^d. \end{aligned} \tag{34}$$

The next to be determined is the length l_j^n of vectors $\overrightarrow{D_j^n O_{jt}^{n,1}}$ and $\overrightarrow{D_j^n O_{jt}^{n,2}}$, which are the sides of the respective right triangles $D_j D_j^n O_{jt}^{n,1}$ and $D_j D_j^n O_{jt}^{n,2}$. Given (20), the solution is:

$$l_{jt}^n = \sqrt{d_t^2 - \left\|\overrightarrow{D_j D_j^n}\right\|^2} = \sqrt{d_t^2 - \left(\left(x_j^{d,n} - x_j^d\right)^2 + \left(y_j^{d,n} - y_j^d\right)^2\right)} = \sqrt{d_t^2 - \left(d_j^n\right)^2}. \tag{35}$$

Note that when $\left\|\overrightarrow{D_j D_j^n}\right\| = d_j^n = 0$ (point D_j is on straight line n), then $l_{jt}^n = d_t$. Moreover, when $\left\|\overrightarrow{D_j D_j^n}\right\| = d_j^n = d_t$ (point D_j is at distance d_t from straight line n), $l_{jt}^n = 0$. Given AAMA set deployment points D_j and air raid directions n at which air targets can be killed, the effective kill points are determined.

The coordinates of the effective kill segment are determined as the coordinates of the ends of vectors $\overrightarrow{D_j^n O_{jt}^{n,1}}$ and $\overrightarrow{D_j^n O_{jt}^{n,2}}$, with:

$$\overrightarrow{D_j^n O_{jt}^{n,1}} = l_{jt}^n \cdot \mathbf{w}_n = l_{jt}^n \cdot \left[w_x^n, w_y^n\right] = \left[l_{jt}^n \cdot w_x^n, l_{jt}^n \cdot w_y^n\right] = \left[l_{jt}^n \cdot \cos\beta_n, l_{jt}^n \cdot \sin\beta_n\right], \tag{36}$$

$$\begin{aligned} \overrightarrow{D_j^n O_{jt}^{n,2}} &= -l_{jt}^n \cdot \mathbf{w}_n = -l_{jt}^n \cdot \left[w_x^n, w_y^n\right] = \left[-l_{jt}^n \cdot w_x^n, -l_{jt}^n \cdot w_y^n\right] = \\ &= \left[-l_{jt}^n \cdot \cos\beta_n, -l_{jt}^n \cdot \sin\beta_n\right]. \end{aligned} \tag{37}$$

Hence, the coordinates of these points are derived: $O_{jt}^{n,1} = (x_{jt}^{o,n,1}, y_{jt}^{o,n,1})$ and $O_{jt}^{n,2} = (x_{jt}^{o,n,2}, y_{jt}^{o,n,2})$

$$x_{jt}^{o,n,1} = x_j^{d,n} + l_{jt}^n \cdot w_x^n = x_j^{d,n} + l_{jt}^n \cdot \cos\beta_n, \\ y_{jt}^{o,n,1} = y_j^{d,n} + l_{jt}^n \cdot w_y^n = y_j^{d,n} + l_{jt}^n \cdot \sin\beta_n \tag{38}$$

$$x_{jt}^{o,n,2} = x_j^{d,n} - l_{jt}^n \cdot w_x^n = x_j^{d,n} - l_{jt}^n \cdot \cos\beta_n, \\ y_{jt}^{o,n,2} = y_j^{d,n} - l_{jt}^n \cdot w_y^n = y_j^{d,n} - l_{jt}^n \cdot \sin\beta_n \tag{39}$$

Among $O_{jt}^{n,1} = (x_{jt}^{o,n,1}, y_{jt}^{o,n,1})$ and $O_{jt}^{n,2} = (x_{jt}^{o,n,2}, y_{jt}^{o,n,2})$, the point closest to the foreseen point of enemy air target emergence near the protected area should be chosen; this selection provides the earliest feasible time for a AAMA set to respond to the air target. Hence, vectors are determined that bind the protected area centre S with all points which constitute the ends of the effective kill segment, $O_{jt}^{n,1} = (x_{jt}^{o,n,1}, y_{jt}^{o,n,1})$ and $O_{jt}^{n,2} = (x_{jt}^{o,n,2}, y_{jt}^{o,n,2})$ (if any), and their scalar products with vector $\boldsymbol{w}_n$.

The vectors binding the protected area object centre S with all points which constituted the ends of the effective strike section have the following form:

$$\overrightarrow{SO_{jt}^{n,1}} = \left[x_{jt}^{o,n,1} - x_s, y_{jt}^{o,n,1} - y_s \right] \\ \overrightarrow{SO_{jt}^{n,2}} = \left[x_{jt}^{o,n,2} - x_s, y_{jt}^{o,n,2} - y_s \right], \tag{40}$$

whereas the scalar products of the vectors with vector $\boldsymbol{w}_n = (w_x^n, w_y^n)$ are described with the formulas:

$$\begin{aligned} d_{jt}^{o,n,1} &= \left(\overrightarrow{SO_{jt}^{n,1}} | \boldsymbol{w}_n\right) = -[(x_{jt}^{o,n,1} - x_s)w_x^n + (y_{jt}^{o,n,1} - y_s)w_y^n] = \\ &= -[(x_{jt}^{o,n,1} - x_s)\cos\beta_n + (y_{jt}^{o,n,1} - y_s)\sin\beta_n] \\ d_{jt}^{o,n,2} &= \left(\overrightarrow{SO_{jt}^{n,2}} | \boldsymbol{w}_n\right) = -[(x_{jt}^{o,n,2} - x_s)w_x^n + (y_{jt}^{o,n,2} - y_s)w_y^n] = \\ &= -[(x_{jt}^{o,n,2} - x_s)\cos\beta_n + (y_{jt}^{o,n,2} - y_s)\sin\beta_n]. \end{aligned} \tag{41}$$

Vectors $\overrightarrow{SO_{jt}^{n,1}}$ and $\overrightarrow{SO_{jt}^{n,2}}$ are parallel to vector $-\boldsymbol{w}_n = -(w_x^n, w_y^n)$ whereas their directions can be the same or opposite to the direction of vector $-\boldsymbol{w}_n = -(w_x^n, w_y^n)$. Hence, the scalar products (41) can be:

— negative, if the directions of the vectors are opposite;
— positive, if the directions of the vectors are the same.

Negative values in (41) meen that the points determined at the ends of the effective kill segment are located at the side of the protected area object centre opposite to the foreseen point of enemy air target emergence (the air target in Figs. 2a and 2b is shown by vectors $\boldsymbol{w}_n$).

Values $\left|d_{jt}^{o,n,1}\right|$ and $\left|d_{jt}^{o,n,2}\right|$ are, respectively, equal to the lengths of vectors $\overrightarrow{SO_{jt}^{n,1}}$ and $\overrightarrow{SO_{jt}^{n,2}}$. Among the two ends of the effective kill segment (represented by points $O_{jt}^{n,1} = (x_{jt}^{o,n,1}, y_{jt}^{o,n,1})$ and $O_{jt}^{n,2} = (x_{jt}^{o,n,2}, y_{jt}^{o,n,2})$) a point is chosen, the scalar product value of which (41) is higher. Hence the larger of the two determined values (41) is substituted in matrix $\boldsymbol{Z}$:

$$z_{jnt} := \max\left\{ d_{jt}^{o,n,1}, d_{jt}^{o,n,2} \right\}, \quad j = 1,\ldots,J, \quad n = 1,\ldots,N, \quad t = 1,\ldots,T. \tag{42}$$

5. Selection of the best AAMA set deployment points

5.1. Function of permissible pair assessment: the number of the permissible AAMA set deployment point and the number of the AAMA set type

Choosing the best deployment points for a defined number of AAMA sets means choosing those pairs with two in each: (i) the number of the AAMA set deployment point from the set of permissible AAMA set deployment points, and (ii) the number of the type of the AAMA set assigned to that AAMA set deployment point to maximize a certain criterion function (of assessment) and maintain the quantities of the sets of selected targeting channels not above the maximum quantities of the AAMA sets of specific types.

The AAMA set deployment assessment criterion indices [1, 2, 3, 4] or the WTA assessment criterion indices [5, 6, 7, 8] shown in the reference literature use combat models based on air target kill probability and the kill probability of the protected object and its air defense system. This paper proposes a different approach to the development of an assessment index for AAMA set deployment plans. It was assumed that this assessment function should consider two factors:

— it is prudent to choose the AAMA set deployment points at which the distance between the air target effective kill points and the protected area centre is the highest (an assumption is made that the earlier an enemy air target is fired upon, the lower the probability of success of that mission for that air target; moreover, the air defense system can potentially repeatedly strike the air target);

— the choice of the AAMA set deployment point should include the kill effectiveness of the targeting channel located there;

— the choice of AAMA set deployment points should include the expected probability distribution assigned to specific air raid directions.

These requirements are met by the following function, which selects the pair of AAMA set deployment points and the targeting channel type:

$$d^o(j,t) = \begin{cases} \sum\limits_{\substack{n=1 \\ z_{jnt} \neq -\infty \\ p_n > 0}}^{N} s_t p_n z_{jnt} & \exists n = 1,\dots,N \; z_{jnt} \neq -\infty \\ -\infty & \forall n = 1,\dots,N \; z_{jnt} = -\infty \end{cases} \qquad j \in \mathbf{D}, t \in \{1,\dots,T\}. \tag{43}$$

Note that when a targeting channel at a specific AAMA set deployment point can not kill an air target incoming from a specific air attack direction, then, given (22) $z_{jnt} = -\infty$ and the formula (23), that specific air raid direction is not included.

5.2. Selection of the best deployment points and the AAMA sets deployed there

The following section presents a formulation of the problem of optimising the deployment point selection and the AAMA set types assigned to them.

$\boldsymbol{V} = [v_{jt}]_{J \times T}$ is a matrix of binary decision variables, interpreted as follows:

$$v_{jt} = \begin{cases} 0 \; - \; \text{no AAMA set of type } t \text{ to be deployed at point no. } j, \\ 1 \; - \; \text{deploy AAMA set of type } t \text{ at point no. } j. \end{cases} \tag{44}$$

The solution to this optimisation problem is to find a matrix $\boldsymbol{V}^* = [v^*_{jt}]_{J \times T}$ which maximizes the function $f(\boldsymbol{V})$

$$f(\boldsymbol{V}^*) = \sum_{\substack{j \in \mathbf{D} \\ d^o(j,t) \neq -\infty}} \sum_{t=1}^{T} v^*_{jt} \, d^o(j,t) = \max_{\mathbf{V} \in \Omega} \sum_{\substack{j \in \mathbf{D} \\ d^o(j,t) \neq -\infty}} \sum_{t=1}^{T} v_{jt} \, d^o(j,t) \tag{45}$$

given the constraints

$$\Omega = \left\{ \boldsymbol{V} = [v_{jt}]_{J \times T} : v_{jt} \in \{0,1\} \wedge \forall j \in \boldsymbol{D} \sum_{t=1}^{T} v_{jt} \le 1 \wedge \forall t \in \{1,\dots,T\} \sum_{j \in \boldsymbol{D}} v_{jt} \le K_t \right\} \tag{46}$$

whereas K_t is the maximum number of the AAMA sets of type number t (t=1,...,T) which can be used in automatic planning of AAMA set deployment.

6. Targeting channel deployment algorithm

The solution to the problem (45) and its constraints (46) is provided by the following algorithm of AAMA set deployment.

1. Determination of the protected area object centre $\boldsymbol{S} = (x_s, y_s)$ with the formula (5) or (6).
2. For each air raid direction n, conversion of the air raid azimuth angles α_A^n (expressed in degrees) to the air attack angles β_n expressed in radians and determined relative to the horizontal axis (OX) of the local rectangular coordinate system according to the formulas (10), (11) and (12).
3. Determination of the air raid direction unit vectors $\boldsymbol{w}_n$ (with formulas (13) and (14)).
4. Determination of the permissible AAMA set deployment points at which AAMA sets can kill air targets in specific air raid directions according to formulas (16)-(22).
5. Determination of the air target effective kill points for the AAMA sets of specific types deployed at those points where killing air targets is feasible ($z_{jnt} = 1$) according to formulas (23)-(39).
6. Determination of the vectors binding the protected area centre $\boldsymbol{S}$ to all points constituting the ends of the air target effective kill segment and the scalar products of these vectors with vector $\boldsymbol{w}^n$ according to formulas (40) and (41).
7. Determination of the value of the assessment function for the following permissible pairs: the permissible AAMA set deployment points and the AAMA set type number, according to formulas (42) and (43).
8. Selection of the best deployment points and the AAMA set types deployed there, according to formulas (44)-(46).

The matrix $\boldsymbol{V}^* = [v_{jt}^*]_{J \times T}$ for the optimization task with constraint Ω can be determined iteratively in

$$LK = \min\left\{K_1 + \ldots + K_T, J\right\} \tag{47}$$

iterations.

1. Iteration 1

   1a. $\boldsymbol{D}^* = \boldsymbol{D}$.

   1b. Find $j_1^* \in \boldsymbol{D}^*$ i $t_1^* \in \left\{1, \ldots, T\right\}$, so

   $$d^o(j_1^*, t_1^*) = \max_{\substack{j \in D^* \\ t \in \{1,\ldots,T\} \wedge K_t > 0 \\ d^o(j,t) \neq -\infty}} d^o(j,t)$$

   1c. $v_{j_1^* t_1^*} := 1;$

1d. $\boldsymbol{D}^* := \boldsymbol{D}^* \setminus \left\{ j_1^* \right\}$;

1e. $K_{t_1^*} := K_{t_1^*} - 1$.

2. Iteration i

2a. Find $j_i^* \in \boldsymbol{D}^*$ and $t_i^* \in \left\{1,...,T\right\}$, so

$$d^o(j_i^*, t_i^*) = \max_{\substack{j \in \boldsymbol{D}^* \\ t \in \{1,...,T\} \wedge K_t > 0 \\ d^o(j,t) \neq -\infty}} d^o(j,t)$$

2b. $v_{j_i^* t_i^*} := 1$;

2c. $\boldsymbol{D}^* := \boldsymbol{D}^* \setminus \left\{ j_i^* \right\}$;

2d. $K_{t_i^*} := K_{t_i^*} - 1$.

3. Iteration *LKC* (the last one)

3a. Find $j_{LK}^* \in \boldsymbol{D}$ and $t_{LK}^* \in \left\{1,...,T\right\}$, so

$$d^o(j_{LK}^*, t_{LK}^*) = \max_{\substack{j \in \boldsymbol{D}^* \\ t \in \{1,...,T\} \wedge K_t > 0 \\ d^o(j,t) \neq -\infty}} d^o(j,t)$$

3b. $v_{j_{LK}^* t_{LK}^*} := 1$.

7. Example application of the algorithm

This section shows the results of implementing the algorithm discussed so far to plan the air defense of an airfield (the reference photo originates from Google Maps), with a single AAMA set having a kill range of 6 kilometres.

The following examples have the following assumptions:

1. The protected area is circumscribed by a polygon with 5 vertices, shown as squares in Figs. 3a-3e.
2. The number of potential AAMA set deployment points is 14.
3. The locations of potential AAMA set deployment points is shown by the small circles in Figs. 7 to 11.
4. The maximum number of AAMA sets is 5.
5. There are 4 air raid directions, with the azimuth and probability values listed for 5 different air raid configurations in Table 1.

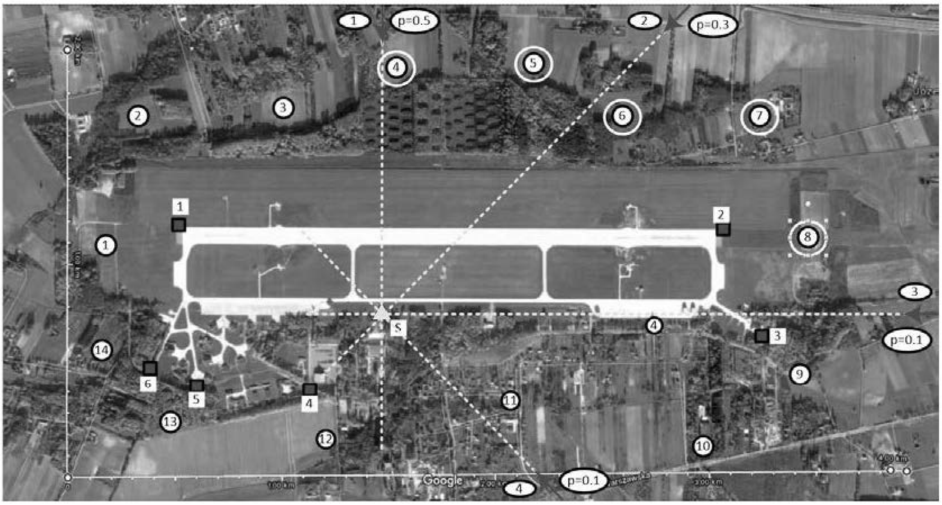
A selection of the AAMA set deployment points are shown as large white circles in Figs. 3a-3e, and the numbers of the AAMA set deployment points are shown in Table 1 in descending order of the values of the criterion function (43). The figures show that the selected set of AAMA set deployment points moves with the changes in the number of the air raid directions of the highest probability.

Table 1

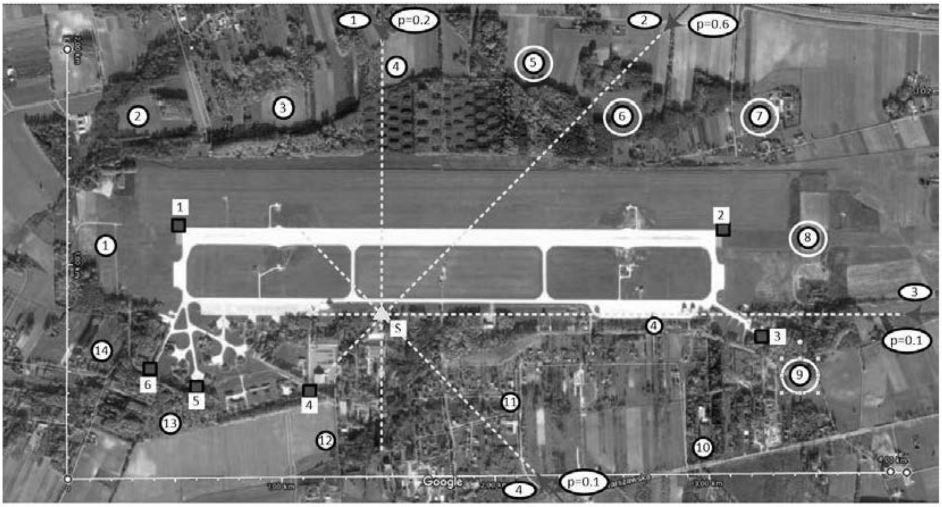
Air raid configuration and calculation results.

		Air raid configurations				
Air raid attributes		Z1	Z2	Z3	Z4	Z5
Azimuth [deg.]	α_1	180	180	180	180	180
	α_2	225	225	225	225	225
	α_3	270	270	270	270	270
	a_4	315	315	315	315	315
Air raid direction probability	p_1	0.5	0.2	0.1	0.05	0.25
	p_2	0.3	0.6	0.1	0.1	0.25
	p_3	0.1	0.1	0.6	0.25	0.25
	p_4	0.1	0.1	0.2	0.6	0.25
Numbering of calculated AAMA set deployment points		7, 5, 6, 4, 8	7, 8, 6, 5, 9	9, 10, 8, 7, 11	9, 10, 8, 7, 11	7, 8, 9, 6, 5

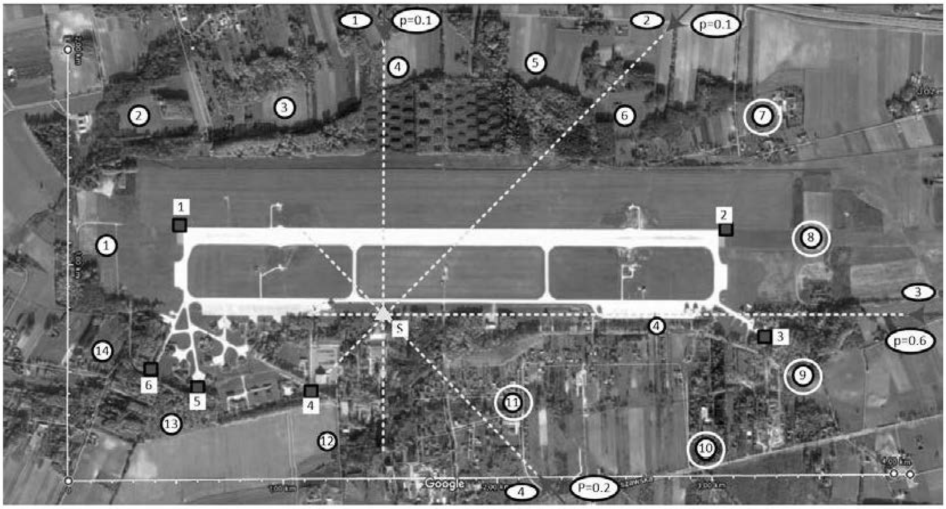
a)



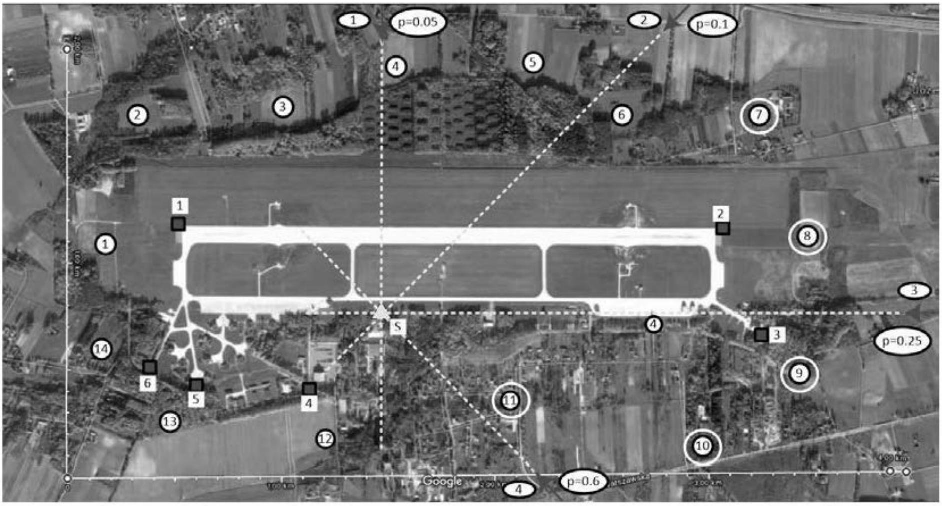
b)



c)



d)



e)

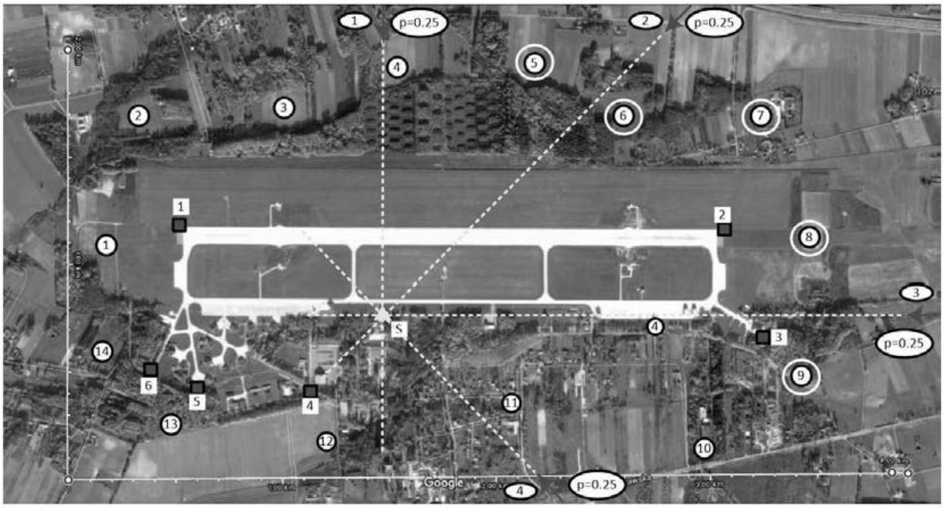


Fig. 3. Illustration of the optimum AAMA set displacement points around the protected area for the airfield under different air raid variants. The air raid variants vary in the probability of air target emergence for specific air raid direction.

8. Conclusion

The paper presents a proposal for a heuristic method of planning the deployment of anti-air missile and artillery sets around installations of high economic or military value. In this paper, a novel approach is argued for the development of the criterion function used to assess AAMA set deployment, as compared to the approach currently in use [1, 2, 3, 4]. The paper shows models of the protected area, its air defense system, air raid, and the decision assessment system. The method for solving the problem presented here is a heuristic one, which provides the determination of sub-optimum solutions. The model presented here can be developed further. More specifically, it would be prudent to add the permitted distance limits between individual AAMA sets and the distribution of air raid combat strength. The method for solving the problem of AAMA set deployment could include methods of binary or genetic programming.

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Planowanie osłony przeciwlotniczej obiektu obszarowego za pomocą środków bardzo krótkiego zasięgu

Streszczenie. W pracy zaprezentowano heurystyczną metodę planowania rozmieszczenia przeciwlotniczych zestawów rakietowo-artyleryjskich bardzo krótkiego zasięgu (VSHORAD) wokół osłanianego obiektu obszarowego. Jako miarę oceny jakości rozmieszczenia przyjęto funkcję zależną od odległości najbardziej oddalonych od środka obszaru bronionego możliwych punktów oddziaływania na cele. Jest to wskaźnik odmienny od powszechnie wykorzystywanego w literaturze wskaźnika bazującego na prawdopodobieństwie przeniknięcia ŚNP przez strefę obrony, prawdopodobieństwie zniszczenia ŚNP oraz prawdopodobieństwie poniesienia określonych strat. Ograniczenia modelu wynikają wprost z ograniczeń nakładanych przez rzeczywisty system obrony powietrznej i naturę obiektów obszarowych. W pracy założono, że system VSHORAD działa w ramach ogólnego systemu dowodzenia i kierowania obrony powietrznej, z którego otrzymuje rozpoznany obraz sytuacji powietrznej oraz plany zamierzeń i zadania bojowe. Przedstawiona metoda została oprogramowana. W końcowej części pracy zaprezentowano wyniki obliczeniowe.

Słowa kluczowe: optymalne planowanie, system obrony powietrznej, osłona przeciwlotnicza obiektów obszarowych, rozmieszczenie przeciwlotniczych zestawów rakietowo-artyleryjskich
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