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Application of parametric modelling methods for estimating the parameters of damped sinusoidal signals*

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Abstract. The paper describes the results of analyses aimed at obtaining information on selected parameters of the deterministic component of a signal being the sum of two waveforms, an exponentially damped sinusoidal waveform and a stochastic waveform.
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1. Background

Estimating the parameters of exponentially damped sinusoidal waveforms is an important problem in numerous scientific disciplines, including telecommunication engineering [10], power engineering [9], system identification [1], and spectroscopy [6]. The reference literature covering the problem is every extensive, including [4] and [5].

This paper describes simulated and experimental tests intended to examine the results obtained from two different methods of data analysis used to determine the angular frequency and the attenuation (damping) rate in sinusoidal signals. Both methods included (i) predictive modelling and (ii) gradient optimization. An assumption used for both tests was that the source of the investigated signal

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was not directly accessible; for a symmetric signal, this meant that the deterministic component included a stochastic waveform.

2. Classic signal frequency analysis methods

Time-domain, statistical, correlational, and frequency analysis methods are all used to investigate signals. If the spectrum of a signal needs to be known, the frequency analysis method is used.

The classic frequency analysis methods applied to continuous signals include the integral Fourier transform and two types of Fourier series, trigonometric and complex. These are popular due to their benefits: they offer both a sound theoretical basis and physical interpretation, with simple relations to time-domain and correlational methods [11].

Numerical frequency analysis of signals has gained high significance in recent years. Its primary tool is the discrete Fourier transform (DFT) algorithm and its numerical implementations known as 'fast Fourier transforms' (FFT). FFT is also the initial point for other signal analysis and processing methods, including cosine and sine transforms, discrete two and multidimensional transforms, adaptive methods, digital filtering, and much more.

3. Parametric methods in the frequency analysis of signals

Spectral analysis tools based on Fourier methods have been both widely investigated and applied. However, their application may be limited sometimes, especially when the signal to be analysed is represented by a vector with a small sample size (e.g. several dozen samples), which provides insufficient frequency resolution for the results.

A feasible alternative in these situations is parametric modelling. The parametric modelling methods are generally based on a relationship between the analysed signal and a certain generative model. Generative models have a variety of uses: they can help identify the primary and secondary frequency responses of a signal, reproduce the deterministic component [1], or enable signal prediction or compression, etc. The initial point for parametrizing a generative model is the assumption that the waveform being analysed is the transient response of a digital filter to a defined input, the spectral density function of which is constant (at a boundary condition). The digital filter transforms, and while this transformation is not a response achieved by physical means, it features attributes which make the response very convenient in different theoretical problems. The input transformation method is defined by factors of transmittance Z of the said discrete system, and these factors represent the discrete system.

Digital filters applied in modelling can be recursive or non-recursive. The choice here depends on the assumptions applied to the type of spectrum of the analysed signal. Absorption spectra are analysed with non-recursive MA (moving average) models. Emission spectra are analysed with recursive AR (autoregressive) models [2].

The linear prediction method is a form of parametric modelling. The concept of presenting the method is based on the initial property of the linear relationship between the samples of a sinusoidal signal. In more precise terms, a sample of the sinusoidal signal can be expressed as a total of an exact subset of its adjacent samples multiplied by certain values. These values are polynomial factors of the numerator or the denominator of transmittance Z of a defined digital filter, which itself is a generative system which outputs the analysed waveform. The wording of the two preceding sentences can be met with some reservation, since it does not formulate the assumptions for the general nature of the signal originating from a certain physical effect (astronomical, geophysical, etc.), an effect that cannot be controlled. It is also not possible to access the signal generated by an object; only a version of the signal is accessible and features a deterministic (information-carrying) component and a stochastic component. Ultimately, the statement is valid that the representation of a signal derived by the application of a predictive (parametric) model is the best representation of the signal's deterministic component, while not being a representation of the signal that is recorded, measured or generated by periodic recording of the readings of certain measuring devices or trends.

The wording of a generally construed spectral analysis is followed by a mathematical description of the analysed signal, complete with the information about its predictive modelling. Putting these two parts in a single section stresses the importance of certain aspects of the non-deterministic nature of the analysed signals and the will of the authors to endow the contents with a better fluency, which might not have been equally possible had the two parts been discussed separately. Later, methods are shown which enable estimation of the parameters of a noisy, exponentially damped sinusoidal signal. Given that the concepts from which these data analysis tools stem differ widely, it seems to be of interest to evaluate their outputs. The results are shown in Sections 7 and 8, while Section 9 contains the final conclusions.

4. Predictive methods of parametric modelling & the model of the analysed signal

An exponentially damped sinusoidal signal was defined as follows:

$$x_d\left[n\right] = A\,\sin\left(\Omega n + \Phi\right)e^{-dn} \tag{1}$$

with: A — initial amplitude of the sinusoidal signal;
Ω — sinusoidal signal angular frequency;

Φ — initial phase of the sinusoidal signal;
d — attenuation rate of the sinusoidal signal;
n — argument of the sine function and the exponential function, also the ordinal number during the sampling period, n = 1,2,3,…

The stochastic signal, used to model the signal noise, was described with this function:

$$x_s[n] = \sum_{i=0}^{N-1} \alpha_i \varepsilon[n-i] \tag{2}$$

with: $\varepsilon\,[n]$ — white noise implementation;
α_i — actual factors;
N — natural number.

The expected value of $\overline{x_s[n]}$ for this signal was zero, whereas the root-mean--square value (variance) was $\sigma^2_{x_s}$.

Ultimately, the analysed signal was converted into the following:

$$x[n] = x_d[n] + x_s[n]. \tag{3}$$

Given the component expressed with formula (1), the following was proven:

$$x_d[n] = a_1 x_d[n-1] + a_2 x_d[n-2] \tag{4}$$

with: a_1, a_2 — model factors (the prediction factors).

The formula (4) implies that the numerical value that described the actual sinusoidal signal (1) sample could be defined by the numerical values of the two previous samples (identified by lower index values) and the 'prediction' factors.

Factors a_1 and a_2 allowed the determination of the angular frequency and the attenuation rate, shown on the right side of formula (1). The determination involved the following formulas:

$$d = \frac{-\ln(a_2)}{2}, \tag{5}$$

$$\Omega = \operatorname{arr}\cos\left(\frac{a_1}{2} e^d\right). \tag{6}$$

However, both are parameters of signal (1). Samples are not directly accessible from it. Samples of signal (3) are accessible. Hence the notation of (5) and (6) should be converted using these formulas:

$$\hat{d} = \frac{-\ln(-\hat{a}_2)}{2}, \tag{7}$$

$$\hat{\Omega} = \operatorname{arr}\cos\left(\frac{\hat{a}_1}{2} e^{\hat{d}}\right) \tag{8}$$

with: $\hat{a}_1, \hat{a}_2$ — prediction factor.

The model parameters were determined by substituting (4) in (3), which gave notation (9).

$$x[n] = \hat{a}_1 x[n-1] + \hat{a}_2 x[n-2]. \tag{9}$$

At least two formulas were required at this point. Both formed a set of equations which could be solved with matrix algebra tools and their numerical implementations. The second equation in the set was defined by using, on the left and right sides of formula (9), a numerical value lower or higher by one than the natural number *n*. In practice, however, sets of equations are implemented with the number of equations higher than the number of the unknowns. In this case, parameters (7) and (8) were derived with a numerical solution of a matrix equation with a rectangular matrix. This procedure usually reduces the effect of the stochastic component on the quality of the results.

When a signal was a superposition of more than one sinusoidal waveform, the total on the right side of equation (4) had twice as many components as the value of the number of sinusoidal waveforms in the analysed signal (1).

This procedure is called the Prony method. Factors $\hat{a}_1$ and $\hat{a}_2$ and were related to the denominator polynomial of a certain transmittance, *H*(*z*), which had the following form:

$$\hat{H}(z) = \frac{\hat{b}_0 + \hat{b}_1 z^{-1}}{1 - \hat{a}_1 z^{-1} - \hat{a}_2 z^{-2}} \tag{10}$$

with: $\hat{a}_1, \hat{a}_2$ — proper estimations of factors a_1 and a_2 in the formula (4);
$\hat{b}_0, \hat{b}_1$ — estimations of the factors in the numerator polynomial of the digital filter transmitter (not included in this description).

The pulse response of the digital filter described by transmittance *Z*, expressed by formula (10), was the model for the signal expressed by formula (1). The signal featured an angular frequency described by (7) and the attenuation rate described by (8). The remaining signal parameters could be derived with matrix equations, which are not shown here.

To improve the characteristics of estimation of (7) and (8), a modification of the procedure described with the formulas (1), (2), (3), (4), (5), (6), (7), (8), (9), (10) could be applied. First, the Prony method was applied to determine the first estimation of the denominator polynomial factors of transmittance (10), i.e.:

$$a^{(1)} = \left[\hat{a}_1^{(1)} \; \hat{a}_2^{(1)} \right]; \quad a^{(1)} = a_{prony} \tag{11}$$

with: $a^{(1)}$ — vector with the denominator factors of the rational function on the right side of the equation (10).

Next, successive approximations of vector $a^{(2)}$, $a^{(3)}$,…, $a^{(p)}$ were derived (p was a natural number). This was achieved with an additional digital filter, which preprocessed the signal associated with each successive iteration. The preprocessing related to the production of the factors of vector $a^{(p)}$ was repeated (p-1) times on the filtered signal. This concept may seem debatable at first, given the risk of amplitude and phase distortions from the successive p-number of signal implementations, the time representation of which could negatively affect the final result of estimating the parameters of signal (1). Note, however, that the next step of the procedure included (i) the waveform filtered in the given iteration and (ii) the modifications of the waveform produced in the previous iteration; ultimately, this allowed the sought numerical values to be produced, which were decreasingly determined by the characteristics of the stochastic component (2). To illustrate certain aspects of the procedure, a simple numerical experiment was done with the objective of presenting the frequency structure of the signals applied in the calculations of successive approximations of vector $a^{(p)}$. A waveform described by formula (3) was used with the deterministic components, being a 0.0249 Hz sinusoidal signal with the stochastic component as an implementation of white noise at a variance 10 times lower than the RMS value of the sine wave. The Welch method [2] was applied to calculate the spectral density estimators of the power of the signals from the successive iterations. The results are shown in Fig. 1, Which shows that the signal described with

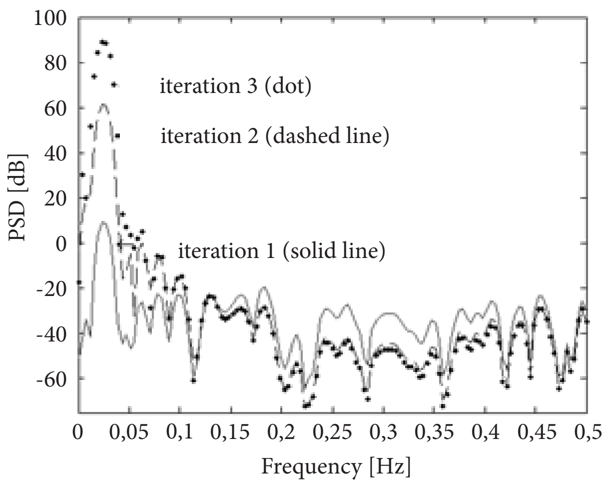

Fig. 1. Result of the spectral density estimation ran on three signals produced by the Steiglitz-McBride procedure.

formula (3) essentially had no noise reduction applied; the signal had its 0.0249 Hz deterministic component (identified by the formula (1)) enhanced. This can be proven by comparing the interval between the frequency representation of the same sinusoidal component and the mean level related to the spectral representation of noise, which was on the same mean level in each plot of the function (iteration 1: 40 dB, iteration 2: 90 dB).

To complete this description of the problem of predictive modelling, note that the iterative approach used here is known as the Steiglitz-McBride method.

Later, a method is discussed which attempts to match the available data to a specific mathematical model.

5. Optimization methods for determining signal parameters

More often than not, the analytical form of the deterministic component of a signal, the parameters of which are being examined, is known to be a certain function. This function can be exponential, multinomial, or trigonometric. It is then rational to determine the parameters of this function by matching it to the available (noisy) data. This matching or adjustment must meet a specific criterion, which is a quality indicator defined by an objective function, which itself is minimized or maximized.

This problem is solved with gradient optimization methods. Both local and global gradient optimization methods exist. This section focuses on local gradient optimization methods, in which the produced point of the solution relates to a defined initial point, the spatial position of which (if a multidimensional generality is assumed) is critical, given the difficulties imposed by the multimodality attributable to the objective function. The spatial position was related to a risk of producing a final solution associated with one of the local minimum values. The values of the function discussed (see formula (12)) near these minimum values could be very different, making the reasoning behind the application of the methods questionable. Hence, what was relevant was the quality of the initial point, the implementation of which required data preprocessing algorithms.

The objective function could have the following form:

$$J(n) = E[(x[n] - x_d[n])^2] \tag{12}$$

with: $J(n)$ — objective function;
$E[.]$ — expected value symbol; all remaining symbols are as defined in formulas (3) and (1), respectively.

The function (12) was minimized due to the parameters of signal (1) which formed the vector (13):

$$w = [A_{est}\ \Omega_{est}\ \phi_{est}\ d_{est}] \tag{13}$$

with all variables analogous to those shown in formula (1); hence, $A \rightarrow A_{est}, \Omega \rightarrow \Omega_{est}$, $\phi \rightarrow \phi_{est}, d \rightarrow d_{est}$.

The vector which defined the initial point of the algorithm was expressed as follows:

$$w^{(1)} = [A_{est}^{(1)}\ \Omega_{est}^{(1)}\ \phi_{est}^{(1)}\ d_{est}^{(1)}] \tag{14}$$

with the variables in superscript being related to the variables present in formula (13).

The numerical values of the coordinates on the right side of equation (14), essential to initialize the calculation process (as relevant to criterion (12)) matching the waveform of (3) to the waveform of (1), were obtained with more or less advanced processing of the analysed signal. The algorithms involved are not presented here, given their relatively simple structure. See references [7] and [8] for a detailed solution to the problem of determining the model of an exponential function for noisy data.

6. Implementation of the Hilbert transform in determining the sinusoidal signal parameters

One of the tools available for a viable solution to the problem of estimating the parameters of the waveforms that this paper concerns was the discrete Hilbert transform. This algorithm was used to implement a complex analytical signal. Its real part was a pre-set signal; the imaginary part was the Hilbert transform of the algorithm. The obtained waveform allowed demodulation of the actual signal amplitude, and determination of the angular frequency and the attenuation rate of signal (1). The details of this data analysis tool are not presented here; they are available in numerous references [11].

7. Simulated test results

This section shows the results of the simulated and experimental tests.

The objective of the simulated tests was to determine the following: angular frequency Ω, attenuation rate d, initial amplitude A, and initial phase Φ of signal (1) when the additive waveform (3) was accessible and related to the stochastic component. The stochastic component was defined by determining the following parameters in formula (2):

$$N = 8$$

$$a_0 = a_1 = \ldots = a_7 = 1/8 \tag{14}$$

and with the following relationship:

$$\sigma_{x_s}^2 = W_e E_x, \tag{15}$$

with: $W_e = \frac{1}{10}, \frac{1}{100}$; E_x — energy contained in signal (1);
$\sigma_{x_s}^2$ — RMS value (variance) of signal (2).

Parameter W_e was a constant which determined the relation between the RMS values of signals (2) and (1). The parameter was defined to produce an SNR (signal-to-noise ratio) of 10 and 20 dB, respectively.

Table 1

Parameters of a one-component exponentially damped sinusoidal signal expressed with formula (1)

Parameter	Initial amplitude [V]	Angular frequency [rad/s]	Initial phase [°]	Attenuation rate [1/s]
	1.000	0.1497	–Π/6	0.020

A single numerical experiment was used to generate 64 transient waveforms described by formula (3). The deterministic component of each waveform was given by formula (1), with its parameters being shown in Table 1. Two 64-element sets of signals were used. Each set had 128 samples and was related to a specific value of parameter W_e. A single waveform (3) produced a 4-element vector of estimators of the parameters for the deterministic component of (1), i.e. initial amplitude A, angular frequency Ω, initial phase Φ, and attenuation ratio d. The vector was expressed as follows:

$$w_{est} = [A_{est} \Omega_{est}\ \Phi_{est}\ d_{est}] \tag{16}$$

with: A_{est} — parameter A estimator;
Ω_{est} — parameter Ω estimator;
Φ_{est} — parameter Φ estimator;
d_{est} — parameter d estimator.

The parameters of signal (1) were identified with a Steiglitz-McBride algorithm (algorithm 1), discussed in Section 4, and a gradient optimisation algorithm (algorithm 2), discussed in Section 5. The deterministic component of (1) was reproduced with the parameter vector expressed with (16). The reproduced signal had a form determined by formula (17).

$$x_o[n] = A_{est} \sin\left(\Omega_{est} n + \Phi_{est}\right) e^{-d_{est} n} \tag{17}$$

with: $x_o[n]$ — reproduced signal, n — sample signal index;
all other parameters are as explained for formula (16).

The results of the simulated tests are shown in Figs. 2 and 3, complemented by Tables 2 and 3. The authors wished to visualise all these results, and the figures are divided into two sections each. Each section features a reference waveform plotted with a bold, black line. The parameters of the reference waveform are known and referential for the results obtained from the identification procedures (algorithms 1 and 3) discussed in Sections 3 and 4. Each waveform has 4 curves grouped in pairs in its vicinity (the yellow curves shown in Fig. 3 are discussed later). The first pair was plotted with dotts. The corresponding reproduced waveforms were generated with the contents of vector (16), which corresponded to the highest ($\Omega_{est,\max}$) and lowest ($\Omega_{est,\min}$) values of Ω_{est}. The other pair of curves, shown with dashed lines, are the exponential waveforms generated with the highest ($d_{est,\max}$) and lowest ($d_{est,\min}$) values of d_{est} that defined the other component of vector (16). The top left corner of each chart features a section with the reference waveform plotted (the bold black line) and all curves reproduced with the contents of vector (16). The latter are shown with thin red lines.

Table 2

List of the statistical parameters in the sets of parameters of signal (1) for two different SNR values *std* — standard deviation; *mean* — mean value from the 64-element set Steiglitz-McBride algorithm (algorithm 1)

Parameter/SNR [dB]	*A* [V]		Ω [rad/s]		*d* [1/s]		Φ [°]	
	mean	std	mean	std	mean	std	mean	std
10	1.0208	0.0704	0.1501	0.003	0.0210	0.003	–30.16	0.096
20	1.0057	0.0389	0.1497	0.001	0.0201	0.001	–30.06	0.047

Table 3

List of the statistical parameters in the sets of parameters of signal (1) for two different SNR values *std* — standard deviation; *mean* — mean value from the 64-element set Optimisation algorithm (algorithm 2)

Parameter/SNR [dB]	*A* [V]		Ω [rad/s]		*d* [1/s]		Φ [°]	
	mean	std	mean	std	mean	std	mean	std
10	0.9651	0.1861	0.1496	0.003	0.0203	0.002	–29.67	5.0146
20	0.9940	0.0382	0.1497	0.001	0.0200	0.001	–29.73	4.7591

The results shown in Figs. 2 and 3 suggest a similar effectiveness of both identification procedures presented in Sections 4 and 5. A comparison between the visualisations shown in the top right corner section provides some general conclusions. The thin red line plots generated with the parameters of vector (16) reproduced the reference signal to an accuracy of a little over 10% (at transient values). The extreme angular frequencies were similar (algorithm 1 to algorithm 2, 0.1436 to 0.1440 rad/s vs. 0.1564 to 0.1570 rad/s); major differences occurred at the fourth decimal place and higher. The maximum angular frequency was 0.0291 s^{-1} from algorithm 1 and 2 0.0257 s^{-1} from algorithm 2. Tables 2 and 3 show that the statistical values calculated for the entire set of estimated

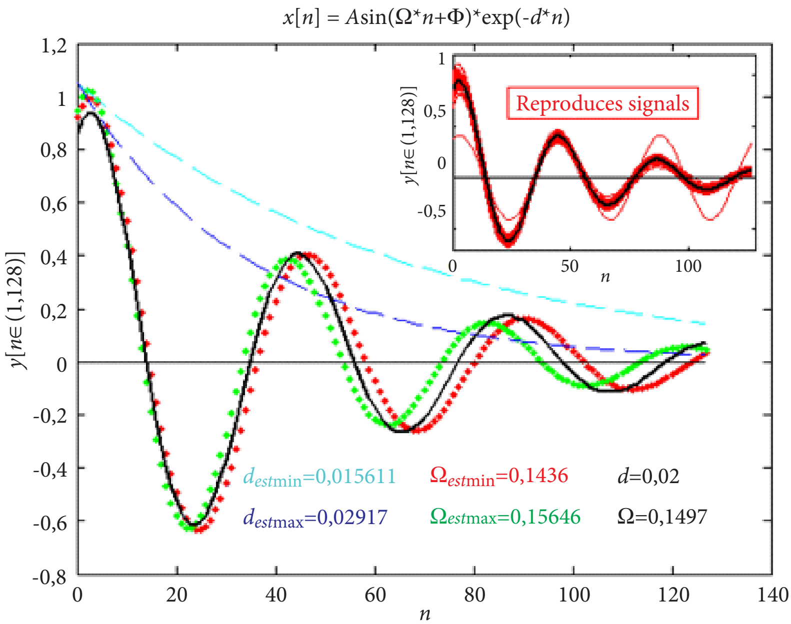

Fig. 2. Results provided by the Steiglitz-McBride algorithm: simulated waveforms.

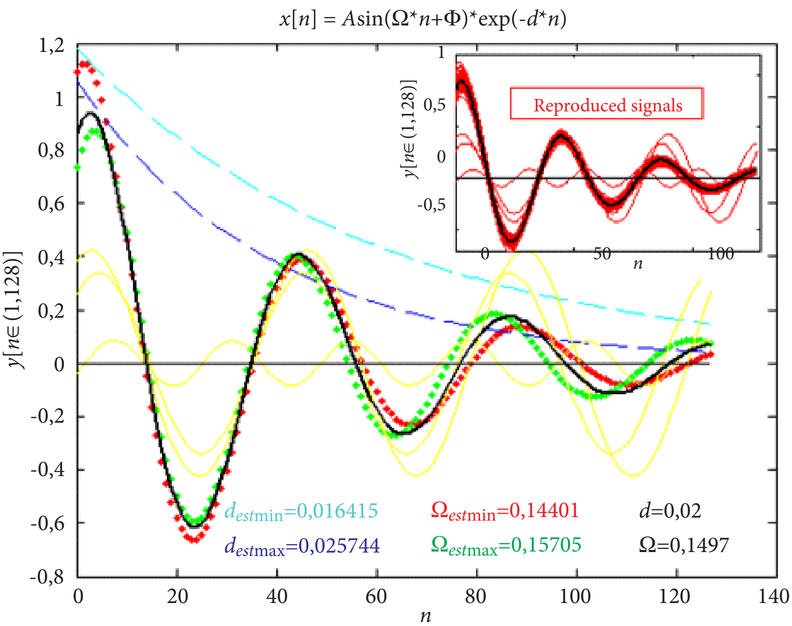

Fig. 3. Results provided by the optimisation algorithm: simulated waveforms.

parameters proved that the parametric model-based procedure was more effective. The highest noticeable differences applied to the mean initial amplitude value, A. This is especially evident for SNR = 10 dB (0.9651 to 1.0208 V) and standard deviation (0.0704 to 0.1861 V). A standard deviation this high was caused by the inclusion of the initial amplitude values in the set, plotted with the solid yellow line in Fig. 3. The solid yellow plot lines represent the non-damped waveforms at a maximum value between 0.1 and 0.5 V. In this case, optimisation failed, and the resulting solution was associated with vector (16) for which the objective function (12) reached one of the local minimum values, which were ineffective at effectively completing the parameter estimations of the component of (1).

8. Experimental test results

This section of the paper presents the results of the experimental tests. The objective of the experimental tests was to determine the frequency of vibration present in a system comprising a power transformer rated at 5 kVA, connected to a capacitive resistance load ($R = 27\ \Omega$, $C = 10\ \mu F$). The steady state condition of this simple power system was disturbed by connecting it for a few milliseconds to a thyristor system, which applied an additional resistance load. This resulted in a distorted current with an amplitude of 150 A, the presence of which in the power system initiated a transient state and sine vibrations in the voltage waveform, the transient value of which

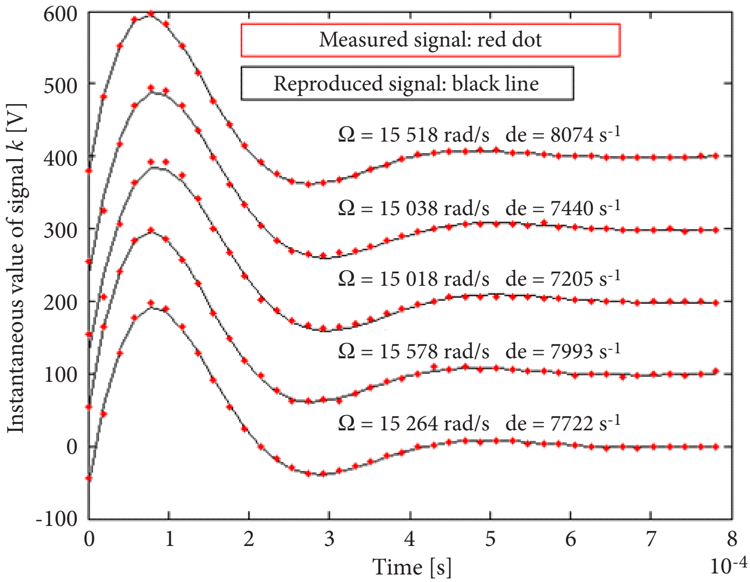

Fig. 4. Results provided by the Steiglitz-McBride algorithm: actual waveforms.

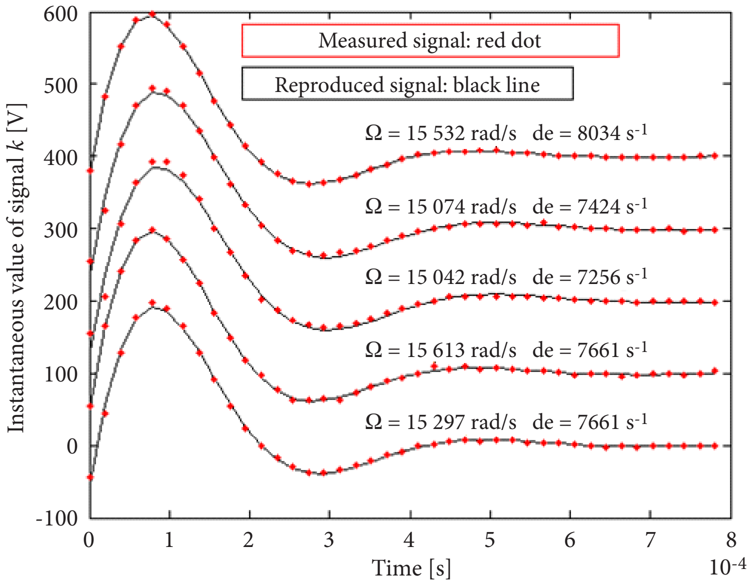

Fig. 5. Results provided by the Steiglitz-McBride algorithm: Actual waveforms.

was recorded at a sampling rate of 51.2 kHz. The objective here was to determine the angular frequency and attenuation rate of the vibration.

The final results are shown in Figs. 4 and 5. The charts represent the recorded waveforms, plotted as dashed red lines, and the reproduced waveforms related to the parameters determined with the identification procedure. The figure shows 5 pairs of waveforms, 10 waveforms in total. As the authors wanted to show this number of curves in a single graphic window, a direct component was added to 4 of the curves to prevent overlapping of the plots.

Given the charts and the numerical values shown in Figs. 3 and 4, the vibration angular frequency was nearly 15 kHz, which corresponded to a frequency response of approx. 2.4 kHz. The mean angular frequency value was 15 283 rad/s for algorithm 1 and 15 311 rad/s for algorithm 2 (with a relative error of 0.2 %), whereas the respective attenuation rate values were 78 687 s^{-1} and 7690 s^{-1} (with a relative error of 0.04%). The visualized results for the specific pairs of waveforms provides a satisfactory compliance with the criterion of time-domain matching of the individual waveforms in each pair.

9. Conclusions

The objective of the tests discussed here was to compare the quality of the results determined using two algorithms to process an exponentially damped sinusoidal signal. The results applied to the parameter estimation quality of a deterministic signal. An assumption was adopted for these tests, that the discrete transient implementation of the deterministic signal was inaccessible, whereas the additive signal was accessible. This approach was justified by measurement practice, which means the preconditions imposed by the specifics of recording a waveforms with measurement noise. These conditions occur in actual research and testing in many fields of science, including astrophysics, medical diagnostics, materials engineering, chemical engineering etc.

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P. FIGOŃ

Zastosowanie metod modelowania parametrycznego do estymacji parametrów sygnałów sinusoidalnych tłumionych

Streszczenie. W artykule opisano wyniki analiz, których celem było uzyskanie informacji o wybranych parametrach składowej deterministycznej sygnału będącego sumą dwóch komponentów — sinusoidalnego tłumionego wykładniczo oraz stochastycznego.

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