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Numerical validation of a method determining thermal diffusivity based on a measurement of a temperature profile

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Abstract. The knowledge of dynamic thermal properties of building elements is necessary to investigate temperature and heat flux changes in natural daily and annual cycles. The basic dynamic parameter is thermal diffusivity. A method for determining its value for real objects has been proposed. This method is based on measuring the temperature in the element's volume and on assuming that the obtained results meet the Fourier equation. Validation by a numerical experiment was made. The wall of the building with known thermal parameters was assumed and the temperature distribution was calculated over time in the process of non-stationary heat exchange. From the results, the diffusivity value was calculated and compared with the data entered into the model. Validations were performed for several accuracy of the temperature value and for two forms of function which approximated the temperature values obtained from calculation. A preliminary analysis of errors has been carried out.
Keywords: measurements of thermal diffusivity, temperature distribution in a building element, approximation, heat transfer
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1. Introduction

To solve the problems of transient heat exchange, it is often necessary to know the thermal diffusivity of the material. This is the basic parameter occurring in the Fourier equation. In construction, the solution to this equation is important in the study of the indoor climate of the building, the selection of heating and air conditioning equipment, and determining the length of the heating season. There are methods to measure this material property, but they do not always give exact results in the case of large mass and dimensions of elements that are building partitions. The majority of diffusion measurement methods consist in comparing the variability over time of the temperature distribution measured and calculated from the Fourier equation. An overview of these methods is provided in [1] [2] [3]. Not all of them can be used for any material because of its physical limitations, for example the melting point. The main limitations of these methods are the necessity to accurately model the boundary conditions that in some cases are a certain idealization of reality. An example may be conditions involving a sudden change in temperature, or assuming a constant heat transfer coefficient [4]. The paper presents the method of determining thermal diffusivity on the basis of temperature measurements at several depths in the wall in a transient state of heat exchange. The heat flow is caused by natural conditions and no additional source is required. The algorithm of a procedure consists in approximation of the results of temperature measurement by means of a polynomial of two variables in time and space. Next, it was assumed that this polynomial fulfils the equation of non-stationary heat conduction and diffusivity was determined. Because we only use the temperature inside the wall, boundary conditions are not important. This method requires measuring the temperature in the interior of the wall, however, in the case of in situ measurements the drilling of appropriate openings is easy.

2. Physical model

The Fourier equation for transient heat transfer is written in the form [5]

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2}. \tag{1}$$

The essence of the method is the assumption of a certain function $T(x, t)$ which is the solution of equation (1). This function is not obtained by solving this equation but chosen in the arbitrary form. It can be, for example, a polynomial (2)

$$T(x,t) = p00 + p10x + p01t + p20x^2 + p11xt + p02t^2. \tag{2}$$

By determining the appropriate derivatives, with respect to time and space, and substituting it in (1), thermal diffusivity can be expressed as

$$a = \frac{p01 + p11x + 2p02t}{2p20}. \tag{3}$$

Formula (3) means the variability of calculated diffusivity in time and space. There is therefore a problem of interpretation of these changes, but it is not the subject of this article. It can also be assumed that the solution of equation (1) is in the form

$$T(x,y) = p00 + p10x + p01t + p20x^2. \tag{4}$$

Hence:

$$a = \frac{p01}{2p20}, \tag{5}$$

and the determined value of diffusivity in this case does not depend on spatial or temporal coordinates. The coefficients $p01, p10, p01, p20, p11$, and $p02$ in equations (2) and (4) are obtained from the approximation of the temperature measurements in the sample. It is possible to approximate functions in different lengths of time. The subject of this article is to examine the influence of the type of the approximation function and the accuracy of the temperature measurement on the determined diffusivity value. This problem was examined by means of a numerical experiment.

3. Numerical validation of the method

In the numerical experiment, the temperature for the partition with the assumed thermal diffusivity was calculated and then the diffusivity was determined according to the presented model. A wall thickness of 0.35 meters has been established with a specific heat of 1000 J/(kgK), a thermal conductivity of 2.5 W / (mK) and a density of 2400 kg/m^3, for these values thermal diffusivity of the wall is

$$a = \frac{\lambda}{\rho c_p} = \frac{2.5}{2400*1000} = 1.04e-6 \quad \frac{m^2}{s}. \tag{6}$$

Equation (1) is solved numerically using the finite volume method [6] in geometry as in Figure 1

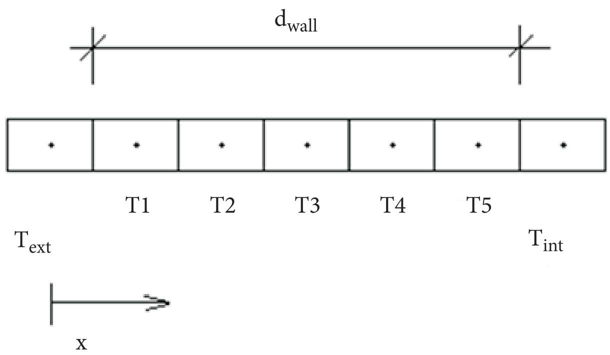

Fig. 1. Geometry and boundary conditions in the wall

One-dimensional heat conduction has been assumed, from the outside and inside wall the air temperature is T_{ext} = 8.6°C and T_{int} = 20°C respectively, the heat transfer takes place with a constant coefficient of h = 12.5 W (m^2K). The wall was divided into five finite volumes whose temperatures were marked from T1 to T5. Distances between the element nodes are 0.07 m. Initial conditions for the time t = 0, the temperature of the entire wall is equal to 20°C. The temperature change was simulated within 48 hours. The calculated temperatures T1 to T5 over time are shown in Figure 2.

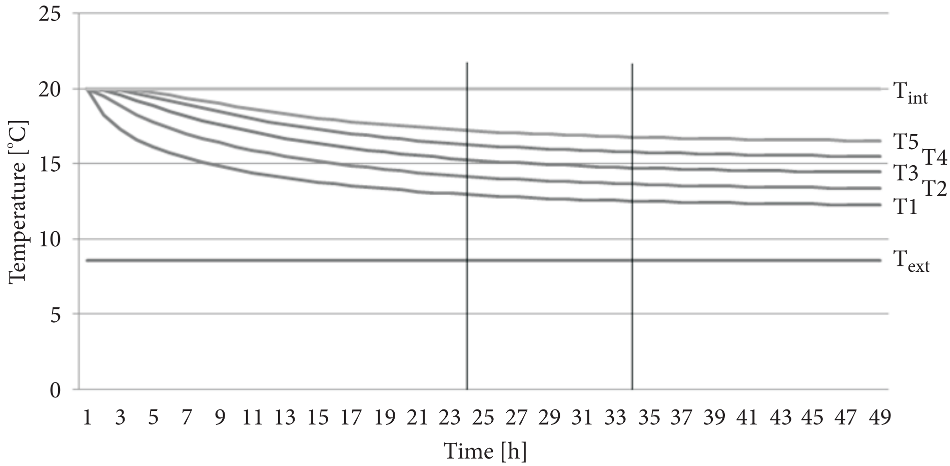

Fig. 2. Variability of temperature in the partition during simulation

A typical temperature distribution is developed in the wall. A ten-hour time interval from 24 to 34 hours selected in vertical lines is selected. The selection of the interval is subjective, it was guided by the fact that it was the time when the typical temperature distribution for the partition is clearly visible and the influence of the initial conditions is not great. Temperature values were calculated with four decimal places. For this interval, the spatial-temporal temperature field approximations of two variables $T(x, t)$ in the form of Eqs. (2) and (4) were made. The time co-ordinate t was defined in seconds, with the range of its variability within the range of -18000 to 18000 assumed. Assuming the approximation function in the form of Eq. (2) by means of the MATLAB environment, the following coefficients were obtained.

Coefficients (with 95% confidence bounds):

$p00$ = 11.43 (11.42, 11.43)

$p10$ = 17.93 (17.85, 18)

$p01$ = −1.238e-05 (−1.27e-05, −1.206e-05)
$p20$ = −6.5 (−6.675, −6.325)
$p11$ = 6.321e-09 (−1.366e-06, 1.379e-06)
$p02$ = 1.383e-10 (1.239e-10, 1.526e-10)

The coefficients have been substituted for formula (5), the diffusion equation obtained depends on the *x*-coordinate and time.

$$a = \frac{-1.238e-5+(6.321e-9)x+2*(1.383e-10)t}{2*6.5}. \tag{7}$$

The average value was calculated as a simple arithmetic mean of the value range of formula (7) for *x* in the considered wall thickness (0.07: 0.35) and t of the analysed time (−18000: 18000), it is 9.522E-07.

The relative error is:

$$\delta = \frac{1.04-0.9522}{1.04} = \frac{0.0878}{1.04} = 0.084 \Rightarrow 8.4\%. \tag{8}$$

Assuming an approximation function in form of Eq. (4), it was obtained
Coefficients (with 95% confidence bounds):
$p00$ = 11.44 (11.43, 11.45)
$p10$ = 17.93 (17.82, 18.04)
$p01$ = −1.238e-05 (−1.285e-05, −1.191e-05)
$p20$ = −6.5 (−6.759, −6.241)
$p11$ = 6.321e-09 (−2.024e-06, 2.037e-06)

Thermal diffusivity obtained from formula (5)

$$a = \frac{-1.238e-05}{-2*6.5} = 9.523e-7. \tag{9}$$

The value obtained from formula (9) is very similar to the average value of formula (7), so it can be assumed that the error is the same.

Most of the available temperature measurement devices offer measurement with an accuracy of one digit after the decimal point. To check the usefulness of such results, thermal diffusivity was determined from rounded results to two and one decimal place, as well as to the whole number. The results are summarized in Table 1.

Table 1

Received values of diffusivity and errors for various approximation functions and accuracy

Assumed solution function	Number of decimal places	Diffusivity	Error [%]
(2)	4	9.522E-07	8.4
(4)	4	9.523E-7	8.4
(2)	2	9.55261E-7	8.1
(4)	2	9.5771E-7	8.2
(2)	1	9.4159E-7	9.8
(4)	1	1E-6	3.8
(2)	0	5.4368E-7	48
(4)	0	2.17596E-6	109

4. Conclusions

The presented method does not impose high requirements in application. Devices for generating thermal impulses that would be difficult to use when testing large objects are not necessary. The results obtained do not differ with changes in the accuracy of the temperature measurement results. The accuracy of 0.1°C, offered by standard meters, is sufficient to determine the diffusion coefficient with an error of less than 10%. A similarly assumed form of the function approximating the measurements does not significantly affect the accuracy of the solution within the limits of applicability of the method. The results obtained are valid for a given temperature course. To generalize, the validation of the method during tests is needed to make real measurements where the variability of temperature is expected in accordance with the daily cycle.

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Numeryczna walidacja metody wyznaczania dyfuzyjności cieplnej bazującej na pomiarze profilu temperatury

Streszczenie. Znajomość dynamicznych własności elementów budynku jest konieczne do określania zmian temperatury i strumieni ciepła w naturalnych dziennych i rocznych cyklach. Podstawowym parametrem dynamicznym jest dyfuzyjność cieplna. Zaproponowano metodę wyznaczania tej wartości dla rzeczywistych obiektów budowlanych. Metoda bazuje na pomiarze temperatury w objętości obiektu i założeniu że wyniki pomiaru spełniają równanie Fouriera. Przeprowadzono walidację metody za pomocą eksperymentu numerycznego. Symulowano ścianę budynku o znanych parametrach cieplnych i wyznaczono zmienność temperatury w nieustalonym stanie wymiany ciepła. Z otrzymanych wyników obliczono dyfuzyjność cieplną i porównano z danymi wprowadzonymi do modelu. Walidacja została przeprowadzona dla kilku dokładności otrzymanych wyników i dwóch postaci funkcji aproksymującej wartości temperatury otrzymanych z obliczeń. Przeprowadzono również podstawową analizę błędu.

Słowa kluczowe: pomiary dyfuzyjności cieplnej, rozkład temperatury w elemencie budowalnym, aproksymacja, wymiana ciepła

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